# Mathematics, Faith and Politics during the Italian Risorgimento
## Snapshot One: Neapolitan Mathematics and the School of Fergola

**Introduction**

The nineteenth century was a time of change in Italy, as well as Europe – change in politics, commerce and industry, in the arts, in religious, philosophical and scientific thinking. Changes also occurred in mathematics including views of what mathematics was and the roles mathematics and mathematicians should play in society. In addition, there were discussions on how mathematics should be taught and the purpose of mathematical education. Institutions for the pursuit and teaching of mathematics sprung up or underwent radical changes. In this three part series of articles we look at how various Italian mathematicians dealt with these changes, especially with regards to how their faith and worldviews provided a lens through which they viewed mathematics and their role as mathematicians in a changing society. This series will also review the outcomes and repercussions of their choices. Their stories can serve as lessons for us as we consider our roles as mathematicians in a changing society.

To accomplish this, in this series we will take snapshots of three different periods in nineteenth century Italian history, focusing on some prominent mathematicians and how they responded to the changes taking place in society. Each snapshot will deal with a critical period during this century when Italy went from being a peninsula divided into states governed by various European powers, to a unified country. This article is the first snapshot of the series and looks at controversies occurring in Naples over what type of geometry should be taught. These disagreements were a direct result of changes occurring in Neapolitan society during the French occupation and subsequent Restoration period, in which the Bourbon monarchy was returned to power. The second article in the series is a snapshot that focuses on the decades between the Restoration and the unification of Italy. This was a period when a national mathematical identity was being created as Italian mathematicians worked to promote Italian mathematics both at home and abroad. This shared goal resulted in collaborative efforts from mathematicians with varying political and religious agendas. The third and final article in the series is a snapshot that looks at the decades when Italy was achieving political unification. Whereas the decades before had been a time of collaboration, political and religious events associated with unification created a divide and mathematicians had to choose on which side of that divide their loyalty rested.

**Snapshot One: Neapolitan Mathematics and the School of Fergola**

In 1839, the leader of the synthetic school of geometry, Vincent Flauti, issued a polemic, reminiscent of the Renaissance public challenges (most famous was Tartaglia and Fiori's over cubic equations) to the analytic school, headed by Fortunato Padula. (Mazzotti, 1998, p. 675) Three geometric problems were proposed, all carefully chosen to guarantee a victory for the synthetic school. The judges were from the Royal Academy of Sciences, men also favorable to the synthetic method. Why then did the analytics take up this challenge, and who were the real victors? To understand this we need to look back to the period in Naples beginning with the Neapolitan Enlightenment and trace the evolution of these two schools, their leaders and their goals both for mathematics and for the future of Naples and Italy.

Synthetic vs. analytic approach to geometry

The synthetic school had formed around Nicola Fergola (1753-1824) a leading Neapolitan mathematician. The work and teaching of the synthetics centered on pure geometry. Its approach to solving problems was specific, i.e. “every problem required a different geometric construction, thus a geometer required intuition along with skill, knowledge, and experience, which could be gained only by long training.” (Mazzotti, 1998, p. 679) The analytic method on the other hand was general. Every problem could be put into an equation. Each problem could then be solved, mechanically, using the same steps. Nothing was left to the intuition of the geometers. This method was seen as ‘easy, ‘mechanical’ and ‘easily learned.’ (Mazzotti, 1998, p. 680)

The analytics and the synthetics both shared the same heritage of mathematics. Their leaders had studied the writings and developments of leading European mathematicians. However, how they chose to view and apply this information varied according to their social, political and religious interests. It led them to hold different ‘images’ of mathematics and to thus practice mathematics in different ways.

Fergola’s early influences

Antonio Genovesi (1712-1769), who held the chair of political economy at the University of Naples, was the primary initiator of the Neapolitan Enlightenment. (Robertson, 2000, p. 27) As a professor of commerce, Genovesi was concerned that his countrymen had fallen behind the leading nations of Europe in their thinking about the material and moral welfare of society. (Robertson, 2000, p. 28) He advocated the cultivation of an educated and informed public, who also had the freedom to express its opinions, which could expose the kingdom’s intellectual life to the issues of human welfare being discussed elsewhere in Europe. He believed Italian rulers could benefit from these new ideas. His objectives were intellectual and educational; they were not immediately political. He saw philosophy as coming to the aid of rulers, indirectly guiding the reforming activity of the government. (Ferraro, 2013, p. 34) The government appeared willing to accept philosophy’s offer of assistance and Enlightenment and reform intersected during the 1770s and 1780s. Some of these reforms took place in the economy, the feudal system, and the power of the Church.

The focus was also on empiricism. Genovesi was concerned with demystifying the various disciplines, including the sciences. He viewed these disciplines as the means for the cultural and economic progress of the country rather than sources of eternal truths. Social problems should take preeminence over theoretical ones. (Mazzotti, 1998, p. 684)

Nicola Fergola was born in Naples. His father wanted him to study law, as this was one of the profitable professions of the time. Fergola studied at the Dominican school, influenced there by the religious values of the Dominicans, including the admonition to live simply. A deeply religious man, he continued with the ascetic values he learned from the Dominicans in his youth.

Fergola attended the University of Naples where Genovesi chaired the department of political economy. His interest lay in mathematics, but as there was little advanced mathematics taught at the university (the main emphasis was medicine and law), he was forced to become mainly a self-taught mathematician. He had access to a private library that contained the works of the leading European mathematicians. His readings encompassed classical Greek texts, almost all the major texts of the $17^{th}$ century including Galileo, Cavalieri, Descartes, Huygens, Newton, Jacob Bernoulli, and all the great writers of the $18^{th}$ century, in particular, Euler, D'Alembert, Daniel Bernoulli and Clairaut. He also studied many works of Lagrange.

Genovesi appreciated the synthetic method and suggested composing science according to the synthetic method and advocated the axiomatic method of Euclid. Also drawn to Euclid's methods, Fergola saw *The Elements* as the epitome of a solid foundation for learning mathematics while also aesthetically pleasing. He proposed to use it as a model for his teaching of calculus and mechanics and also for issues relating to the existence of God and miracles. (Ferraro, 2013, p. 23, pp. 26-27)

Fergola's school

In 1770 (when he was only 17 years old) the *Liceo del Salvatore* (School of the Savior) hired Fergola to teach. The following year he opened his own private studio where he taught advanced mathematics. Such private schools were not unusual in Naples. They provided basic training in various disciplines but often also offered a superior level of preparation for the university and for technical careers. Fergola's school quickly acquired a good reputation and many of the brightest students studied there. One of these students was Annibale Giordano, a mathematical prodigy, who in 1786, at the age of 16, presented to the Royal Academy of Sciences in Naples an article entitled "Continuation of the same subject" which brought him a monthly stipend from the school. The following year he wrote an article on the problem of Cramer, which was published in *Memori of the Accademia* and gave him a certain notoriety throughout Europe. (Ferraro, 2013, p. 47)

During 1770s and 1780s Fergola entertained fairly broad interests. He taught and did research in both synthetic and analytic geometry. He praised the works of Lagrange (a leading proponent of algebraic and analytic mathematical methods) and D'Alembert. Fergola and his school thrived during this period. His school generated most of the scientific production in Naples regarding mathematics. However, by 1786 Fergola's interests begin to shift toward the synthetic method. He showed particular interest in the "forgotten method of the ancients" (Mazzotti, 1998, p. 691) and what he considered their elegant, intuitive and certain solutions. He began to see analytical methods as being unreliable and lacking secure logical foundations.

In 1789 Fergola was appointed full professor at the *Liceo del Salvatore.* That same year his prodigy, Giordano, received a professorship at the Military Academy. However, what looked like the beginning of a promising academic career for Giordano took a radical turn when Giordano joined the reformists to promote Jacobin ideas.

Jacobin Science and mathematics

The Neapolitan Jacobins believed that science should speak to the real needs of the people, a view of science in opposition to that of one "pure" or contemplative. The leaders of this school in Naples were Carlo Lauberg, a scientist, and Giordano. Lauberg had applied for the same professorship as Giordano, but Fergola opposed his appointment because he viewed him as morally reprehensible (Lauberg was a defrocked priest). (Mazzotti, 1998, p. 687) Fergola advocated for the appointment of Giordano instead.

Lauberg and Giordano opened a school and wrote textbooks advocating the analytic method. The analytics presented universal methods that permitted the development of scientific knowledge in the modern era, including the application to the spheres of social and political sciences. They wrote their textbook with the goal of being useful to their country. To them, math was a universal language that every rational being could comprehend. They believed in the mathematization of human sciences and that social and political order could be built on the basis of the new theorems of the social sciences of politics and economics. (Mazzotti, 1998, pp. 689-690)

With the French Revolution in 1789 (and the execution of the king and queen in 1794) the European monarchy's support for reform began to wane. (Robertson, 2000, p. 40) The Revolution provoked a conservative reaction against the ideas of the 18$^{th}$ century European Enlightenment. Since the Enlightenment thinkers had greatly valued science, there was also a reaction against applications of science in various aspects of life because those applications could lead to materialism and ultimately to political radicalism. (Burns, 2011, p. 103) French Revolutionary ideas about religious toleration, civil equality, freedom of the press, and liberal ideas about education clashed with the church's view that it had the responsibility for the spiritual, moral and educational welfare of the people. (O'Leary, 2007, p. 45) And while the governments had been anti-clerical in the 1770s and 1780s, this changed in the 1790s. The Church now became the main ally of the monarchy in its opposition to Jacobin tendencies and liberal ideas.

Political and Religious Changes in the 1790s

During the 1790s Reactionary Catholicism took root in Naples. It aimed at a return to a theocratic society based on an idealized image of the Middle Ages. Its proponents saw themselves as defenders of a venerable and natural conception of the world, with the Church and Crown forming the two pillars of society, and religion seen as the only basis of a 'natural' human society. Reactionary Catholicism was 'reacting' to the rational and often abstract laws of the Universal Reason of the philosophers. These Catholics believed that the natural order of the Church and State had first come under attack during the Renaissance initiating moral and political corruption that led to the Protestant Reformation and later to political revolutions. These conservatives oversimplified much of history, inventing much of this interpretation and their "venerable tradition" in order to provide credibility and authority for their arguments. (Mazzotti, 1998, p. 685-686) The synthetic school, with its emphasis on pure mathematics and suspicion of the applied analytical approach, became the "scientific component" of the Reactionary Catholic movement. Fergola, as leader of the synthetic movement, and his school flourished during this time period.

In 1796 the French armies invaded Italy and the era of the Italian Republics commenced. Though short-lived, this era saw the beginning of societal transformations that would challenge the ancient regime in Italy, whose power lay in the nobility, the religious orders and the papacy. The separate Italian principalities transformed into centralized bureaucratic autocracies. Feudalism was abolished and fiscal and financial administrations were centralized. Vast tracts of Church land were sold to pay for these changes. Privatization of land was promoted to improve agriculture and new industries were developed. These changes accelerated economic development but also heightened the social costs of modernization. (Davis, 2000, p. 7)

In January 1799 a revolutionary agitation shook Naples and brought about the proclamation of a Jacobin republic as well as the abolition of feudalism. Fergola left for the countryside during this time period to avoid the conflict. Six months later the Bourbon allies with the help of the British militarily defeated the Neapolitan Republic. There followed a period of mass executions and exile of the survivors. Many of these victims were young well-educated men and women with some training in the scientific disciplines. Almost a generation of intellectuals and professionals in Naples was destroyed. (Mazzotti, 1998, p. 690)

After the Republican defeat Fergola, a loyalist, returned from his countryside sojourn. The king rewarded the loyal and conservative ecclesiastics and professors by giving them positions of power in dioceses, universities, and military and naval academies. The king also turned to them to reorganize the public education system and cultural institutions. Reformists, on the other hand, were viewed with suspicion and were held responsible for the revolution and for social disorder in general.

By 1800 the preeminence of the synthetic methods became even more pronounced in the teaching and research of Fergola's pupils. Mathematics had become synonymous with synthetic and descriptive geometry (also taught and developed in Fergola's school). Thus, it is ironic that though Fergola's awareness of mathematical developments in Europe and his sharing of this knowledge with his pupils had been responsible for bringing Naples into the European mathematical community, now, with his focus on synthetic geometry and its deemphasizing the role of analytical geometry, popular in the rest of Europe, "Naples, was, once again, out of Europe." (Mazzotti, 1998, p. 691)

Religiosity of Fergola and the Catholic Enlightenment

Fergola's 1789 appointment as full professor at the *Liceo del Salvatore* (School of the Savior) was a royal appointment and came with the condition that he publish his lectures. In 1792-1793 Fergola published his lectures on Newton's *Principia*. These lectures are essentially an apologetic reading of the *Principia*, where Fergola thinks of a God who orders the heavens with geometric laws. (Ferraro, 2013, p. 30) Fergola was particularly interested in the study of forces "for it permits us to discover 'the laws of the universe' and 'the deep knowledge of who rules and sustains it.' Indeed, according to

Fergola, the source of all the forces acting in the universe is the 'Hand of the Living God.'" (Mazzotti, 1998, p. 693)

Fergola also applied his synthetic method to religious themes. In 1804 he wrote *Theory of Miracles* in which he argued for the existence of miracles in general and in particular the miracle of the liquefaction of the blood of St. Januarius, the patron saint of Naples. In it he "argues for the spirituality of the soul and the nonmaterial origin of human thought. He rejects materialism, in its many variants, because empirical reality is too complex to be reduced to material causality." (Mazzotti, 1998, p. 692) He wanted this work to be a geometric discussion of religious matters. (Ferraro, 2013, p. 28)

In his private life, Fergola continued to live as an ascetic. As a leader of a school he saw his responsibility as going beyond teaching mathematics, "to help the pupils in their spiritual growth, in order to make them good Christians and good citizens." He was concerned with his students' moral behavior and their religious and political ideas. "Mathematics, as it was practiced in Fergola's school, was a 'spiritual science,' a science that brings its practitioners very close to the mind of the Almighty. It was also a powerful resource in the fight against atheism and materialism." (Mazzotti, 1998, p. 692)

In his teachings we also begin to understand how he saw the distinction between synthetic and analytic geometry. For Fergola synthetic geometry was the language of God. The true mathematician contemplates the world around him and subsequently the geometrical character through which it is written, in order to understand eternal truths. Synthetic geometry, with its emphasis on intuition and reasoning completely independent of matter, was more certain and more valuable than analytic geometry. The expediency of analytic geometry for applied mathematics made it a useful instrument, but certainty was not attainable in analytic geometry. Certainty was limited to pure mathematics. We also begin to see a portrayal of what a true mathematician is –an ascetic, a contemplator of geometrical truths. He is not concerned with the applications of math to political or social problems. He is part of an ancient tradition that went back to the Middle Ages when the church, the monarchy and mathematics formed a harmonious alignment. The idea that the synthetic school was a continuation of an ancient tradition was actually a myth perpetuated by Fergola. It was, rather, a new phenomenon, a reaction to the secularization of science and society.

Reactionary Catholics used Fergola's writings to argue for the fallible nature of empirical knowledge and the limits of scientific investigation. Fergola's *Theory of Miracles* was written at the bequest of Bishop Colangelo, who was interested in science, philosophy and history. At the same time Fergola was working on this essay, Colangelo also published a book on the progress of science. He attacked the autonomy of scientific practice from religion. "Colangelo claims that only a scientist who is also a good Christian can achieve relevant scientific results." (Mazzotti, 1998, p. 695) (This claim resurfaced in Ventura's 1824 eulogy of Fergola in which Ventura stated that "only a Christian can penetrate mathematical truth and investigate empirical reality properly.") (Mazzotti, 1998, p. 697) Colangelo drew on Fergola's work in *Theory of Miracles* in his propositions regarding how God rules the universe. God is characterized as an absolute monarch whose will is the source of all natural law. A miracle is then an event that breaks some natural law. If God can break his own laws to allow miracles to happen, then cannot the king, if he is the source of sovereignty, also "break" his laws? This argument took on political significance two decades later when King Ferdinando I 'legitimately' abolished the constitution he had granted to his kingdom.

Napoleonic Era

At the beginning of the 19th century, Italy was a collection of principalities under the rule of various European powers, along with the Marches, Umbria, and the Papal States, which were under papal rule. No constitutions protected the rights of individuals. Customs and practices were local, as were weights and measures and currency. The different parts of the country were isolated geographically; there were no railways connecting the different regions. The Church was in charge of the educational system and of ministering to the poor. The ecclesiastics and the aristocracy held power. There was no real middle class. The professional class consisted mainly of lawyers and physicians. All this was to change under Napoleonic rule.

In 1806 Napoleon was crowned King of Italy in Milan. The following year he established his brother, Joseph, as King of Naples. Thus began the decade long period of French rule that was to change the foundations of Italian life and precipitate the unification of the Italian peninsula.

Napoleon had a passion for uniformity and for centralized administration. Decisions previously made at the local level were now made by central authorities. Fiscal and commercial reforms were instituted; weights, measures and currency were standardized and internal customs barriers were removed. The French penal, civil and commercial codes were introduced, the feudal system was abolished and taxation systems were reorganized. The political and economic power of the Church was under attack and many ecclesiastical lands were sold. New religious liberties were granted, with equal rights being given to Jews.

Modernization brought by the French benefited some but not all. Resistance to change was also strong, for to many it meant higher taxes and military conscription. (Duggan, 2008, pp. 8-10) There was also resistance to standardizing weights and measures for how could a plot of mountainous land be considered equal to the same size plot of fertile land? According to Lucy Riall, "This struggle between those who embraced change and those who resisted it, and between the manifest need for reform and the need to control its consequences, was to characterize political debate in Italy during the decades following the fall of Napoleon." (Riall, 2009, p. 10)

First controversies between analytic and synthetic schools

These societal changes were reflected in the mathematical community and led to the first real controversies between the analytic and synthetic schools. At this point Fergola and his students held the leading mathematics chairs in the universities, essentially controlling the teaching of math in the university and also heavily influencing mathematics education in the high schools. But a new system of education and a new type of 'mathematician' began to emerge which not only threatened this dominance but the role mathematics was to play in society.

In 1808 the Royal Corps of Engineers of Bridges and Roads was established. Placed under the Ministry of the Interior, the Royal Corps had jurisdiction over roads, bridges, dams, monuments and public buildings, as well as ports and channels. (Ferraro, 2013, p. 55) It was a modernizing force in service to the central government rather than to the local authorities. The Royal Corps provided new infrastructures, which facilitated economic and technological changes. Its effects were also felt in the educational system, for it taught a different type of mathematics. Engineers now assumed a higher social status on par with professionals in medicine and law. The training of engineers required new institutions of higher education and a different kind of mathematical training.

In 1807 the government issued a decree prescribing that all textbooks used in the public schools had to be approved by the Ministry of Internal Affairs. (Ferraro, 2013, p. 70) A commission was appointed to oversee the compilation of these textbooks. Nicola Fergola served as one of the members of this commission as he had considerable didactic experience in both public and private education. With Fergola's help, the task of drawing up a complete course of mathematics was given to two of Fergola's students, one of which was Vincenzo Flauti (1782-1863). Flauti would become the face of the next generation of Fergola's school. The commission wanted textbooks that would organize all the necessary mathematical knowledge needed by students in secondary schools in a systematic way. It was to be an integrated curriculum in which arithmetic, geometry, algebra, differential and integral calculus, etc., were no longer treated as isolated and independent, but united and connected. For many mathematicians the unity of the course was to be ensured by the notion of abstract or general quantity, from which they built the analysis or algebra. These methods were then applied to other mathematical disciplines, including the geometry. This was the approach of members of the analytic school, closely linked to the ideas of Euler and Lagrange. Others, however, opposed this idea of algebra as a center of mathematics. Fergola and the synthetics wanted to ensure that this unitary presentation of mathematics would be based on the synthesis and the Euclidean method. They endeavored to design a course based on this paradigm. (Ferraro, 2013, p. 71)

In March of 1811 the *Scuola di Applicazione di Ponti e Strade* (School of Application of Bridges and Roads) and in August 1811 the Polytechnic School were created, based on the model of the *Ecole Polytechnique* in France. Both were to provide the basic scientific training for engineers. These schools were structured differently than the universities. Their curriculum was radically different, with focus on the applied mathematics needed for building technological and economic reforms. Entrance was based on an exam designed to test mathematical knowledge.

One disadvantage of the French model of centralized decision-making was that the decision makers often did not have first hand knowledge of local conditions. The government turned to the engineers and the supposed neutrality of their scientific judgments to be their local experts. (Mazzotti, 2009, p. 256) The engineer was seen as speaking the voice of reason, and his understanding of natural and social reality gave him moral superiority in the eyes of the reformers, many of them now returning from exile. Appointment to the Corp of Engineers was based on a merit system of promotion alien to social life in the ancient regime. It opened up opportunities for men from the middle level of the bourgeoisie to acquire a new status within society. However, the rise of the engineer's status and authority in society meant that traditional authorities, e.g., the landed aristocracy, the church, local communities, etc., lost their status as decision-makers. This naturally created feelings of jealously and created social tension between the engineers and the local authorities.

This social tension led to a controversy between the members of Fergola's school who had recently had control of teaching mathematics in the university (and therefore heavily influenced mathematics education in the high schools) and those who were associated with the Corps of Bridges and Roads and the *Scuola di Applicazione*. The controversy was never exclusively academic. (Ferraro, 2008, p. 4) Animosity between the two groups went beyond technical mathematical arguments; there were accusations of "moral depravity" and corrupting the minds of young students. Scientific, educational, political and academic career issues intermingled with the future understanding of how mathematics was to be done. To understand the reason for these polemics, let us look further at the premises behind each of these schools.

The synthetic school believed that mathematics was an intellectual activity practiced to uplift the human spirit. (Ferraro, 2008, p. 111) It did not require practical applications, was not done for technical preparation for any profession. The teaching of mathematics should focus on rigor and deductive reasoning, for these practices would help individuals learn to develop their intellectual capacity, to become inventive and creative. Fergola's school saw analysis as contaminated by empirical considerations and in need of more rigorous foundations. Synthetic teaching was based on discovering and grooming the natural talent of exceptional students. A typical product of this system was the child prodigy. (Mazzotti, 2009, p. 266) (Note: Giordano is an example. To promote the idea that he was a child prodigy, his age was given as 15 rather than 17 when he solved his famous problem in 1886.)

The analytic method was heavily influenced by the great French analysts of the $19^{th}$ century. It supporters were seen as progressives, young 'outsiders' aware of current mathematical developments, and supporters of reform. (Mazzotti, 1998, p. 677) The supporters of the analytic method described it as a natural way of reasoning. This method was based on an assumption that all complex problems could be broken down into their elementary components, which could be solved by techniques of algebraic analysis and calculus using mechanical manipulation of symbols. With proper training any student could master its techniques because it was simply the natural way the human mind worked. (Mazzotti, 2009, p. 263)

The controversies between the two schools began formally in 1810 with a criticism of Fergola's work by Ottavio Colecchi, a Dominican philosopher and teacher of calculus at the *Scuola di Applicazione*. Colecchi reproached Fergola for putting too much emphasis on pure geometry and being dismissive of advances made by modern analysis. (Mazzotti, 1998, p. 677) Colecchi's attack may have been precipitated by the fact that the Fergoliani school at this point had complete dominance of the universities and the Academy of Sciences. Also Flauti had been charged with writing the new math textbooks for the schools and the educational ideas of Fergola and his synthetic methods were certain to permeate these books. In 1811 the *Scuola di Applicazione* was to open, followed by the *Scuola Politecnica e Militare* in 1812. These schools did not fall under the control of the Fergoliani, and their supporters wanted to ensure a different model in teaching and research.

In 1811 Fergola's school responded with the publication of a brochure that had the stated purpose of showing the value its school had in the rise of innovative scientific production, mainly in the school of geometry, and to a lesser extent, in the field of analysis. (Ferraro, 2013, p. 60) It also contained a defense of the cultural value of geometric research conducted with synthetic methods, stating that is was a matter of national pride to develop the ancient geometry. It claimed Neapolitan mathematicians had not had the chance to cultivate research activities, or because of the absence of a suitable cultural climate or too many commitments, had been unable to develop the ancient geometry. But with the emergence of the figure of Fergola and his school, this ignominious void was filled. With the publication of this brochure the controversy finally exploded.

This exchange of diatribes was just the first in a series of accusations between the two schools, which would culminate a quarter century later in the 1839 disfida. Each school responded to changes happening in Naples at the turn of the century. The analytics and the engineers saw mathematics as an instrument of modernization. It brought a hope for technological innovation and a democratization of science itself. Doors were opened to a new bourgeois society made up of equals, who could play a new role in society while earning a living. More conservative mathematicians with strong religious ties, such as Fergola and Flauti, considered the synthetic approach as the most "appropriate response to what they perceived as a broad cultural and moral crisis of European civilization." (Mazzotti, 2009, pp. 23-24)

<u>Bourbon Restoration 1815</u>

In 1814 Napoleon was defeated and the 1815 Congress of Vienna placed most of the Italian States almost entirely under foreign dominance. Austria had control, either directly or militarily, over much of Italy. Piedmont remained the only Italian state with some autonomy from Austria. However, the Restoration governments did not dismantle the excellent administrative machinery Napoleon had left in the Kingdom of Italy. They adopted and adapted many of the French administrative and legal innovations and continued the modernizing efforts set in motion by the French. They recognized the value of new roads, canals and railways for creating trading opportunities. The rulers looked for ways to promote economic growth without making concessions of political or cultural freedoms. The Restoration governments were interested in investing in education, but not in any subjects that could lead to political upheaval. (Clark, 1998, pp. 23-24) Science seemed a safe subject, and the study of science, especially applied science and engineering, acquired a social prestige. (Hearder, 1983, pp. 30-31) The school of Fergola continued to dominate the university and Academy of Sciences, but the *Scuola di Applicazione*, after some initial difficulties, continued to be a major player in Restoration Italy. These difficulties were due to the return to power of those most threatened by engineering activities, i.e. the local political conservatives. The autonomy of the school was restricted. A branch of Bridges and Roads replaced the Corps of Engineers. Staff was reduced and provinces were allowed some local oversight on projects.

Fergola by this time was fading. He suffered from mental illness and his religious conservatism now approached bigotry and a kind of religious delirium. (Ferraro, 2013, p. 75) It was time for the next generation to take the lead. The two main leaders of the Neapolitan mathematical establishment during the years 1820-1840 were both students of Fergola. One was Vincenzo Flauti who now became the successor and interpreter of Fergola. The other was Francesco Tucci (1790-1875). Tucci became the leader of the analytical camp. In 1813 Tucci was appointed a professor at the Polytechnic School and later a professor at the Military College where he became the director. Again, it was an argument over the value of algebra in solving geometric problems that appears to have brought about the break between Tucci and his former teacher. Tucci, as a professor in the *Scuola di Applicazione*, was concerned with concrete problems his engineers faced and ways to train and prepare them to solve those problems. To him, this could be done best using the methods of analysis and algebra.

Flauti at this time continued to work on his textbooks. But he also devoted much time to the publication of the work of Fergola. It became an objective of the school of Fergola to develop a myth around the man Fergola and his influence as a mathematician and a defender of the Christian faith. Fergola was primarily a teacher. While he was a devout Catholic and loyalist, he was temperate in his viewpoints. But with Flauti now at the helm, the synthetic school became more restrictive in its views; Flauti was also more interested in using the synthetic approach in academic battles. (Ferraro, 2013, p. 81) However, in terms of real scientific output, little was being produced by the synthetics during these years. (Ferraro, 2013, p. 84) It seemed all of its efforts were being directed at perpetuating a myth about the importance of the school and in attacking the work of the analytics. On the other hand, with the exception of Tucci, the analysts also had little scientific production. Their focus was using mathematics as a tool for engineering.

In 1820 a revolution took place in Naples that resulted in Ferdinand I pledging to grant his constituents a constitution. However, he then revoked this constitution in 1821 (recall the argument in Fergola's *Theory of Miracles* where a sovereign ruler can override his own laws). (Mazzotti, 1998, p. 696) In this political crisis the *Scuola* became a target for conservative politicians. The director of the school had to defend the unique role of the school for the training of engineers, which was different than the university training required for other mathematicians and scientists.

Fergola died in 1824 and one of his pupils, Ventura, gave an impassioned eulogy stressing Fergola's ascetic life and using him as proof that it was possible to be a good mathematician and a good Christian. He went on to state that only a Christian could penetrate mathematical truth; in the hands of atheists mathematics becomes a tool for the destruction of society.

Carlo Rivera became the director of the *Scuola di Applicazione* in 1826. Rivera worked to restore the authority and autonomy of the Corps. Rivera realized that in order to implement technological and economic improvements throughout the kingdom, there needed to be standardization of weights and measures and a single decimal system. This went against traditional practices of local areas having their own measurements for different kinds of items. Measurement needs differed from region to region. Once again it was centralization versus local autonomy, standardization versus variability. (Mazzotti, 2009, p. 264) Also at stake was tax reform, for the multiplicity of existing local measures offered landowners the ability to evade taxes. (Ferraro, 2013, p. 95)

Ceva Grimaldi, a member of the aristocracy and a future prime minister of the Kingdom of the Two Sicilies, headed up the anti-standardization campaign. Grimaldi defended the interests of the provincial elites, landowners, and private contractors. He repeatedly attacked Rivera and his engineers, calling into question the validity of the training the engineers received at the *Scuola di Applicazione* as well as their accomplishments.

Both sides waged a public relations campaign to win over adherents. Rivera repeatedly extolled the economic and technological accomplishments of his engineers: land reclamation, creation of a modern fishing industry, establishment of factories, development of new villages, etc. He wrote both rhetorical pieces and technical reports extolling the virtues of his engineers and defending their work. In 1835 he organized a special exhibition at the *Scuola di Applicazione* to celebrate the Corps. (Mazzotti, 2002, p. 92) By having his students give lectures on a variety of technical and mathematical topics and by displaying their work, which included plans of bridges and ports, he highlighted the valuable training provided by the school. Rivera also demonstrated the value of the coordinated group effort of the Corps as opposed to the isolated genius, which was characteristic of the synthetic school.

Grimaldi and the conservatives waged their own public relations campaign. Not only did they question the training of the engineers and engineering's ability to solve social problems with their mathematics, they criticized the need for such societal changes. They promoted a mythical description of a happy and utopian Neapolitan countryside, one that didn't need the innovations of the engineers. In a historical essay on public works dating back to the Middle Ages (appeals to the Middle Ages were common by conservative factions throughout the Risorgimento), Grimaldi derided the notion that civilization in southern Italy began with the Corps of Engineers. (Mazzotti, 2009, p. 261) In his description of pre-French Naples, everything functioned well and in harmony – peasants and artisans lived together, cared for by the aristocracy and the church; the roads and waterways were well managed. The enemy was not the king or the church or the existing order, but whatever was 'foreign' or abstract'.

<u>The 1839 Challenge</u>

The controversy between the analytic school (the engineers) and the synthetics (conservatives and Flauti) escalated during the 1830s. Several incidents toward the end of the 1830s precipitated the 1839 challenge. One was the standardization of weights and measures that was scheduled to go into effect in 1840. In 1838 Grimaldi wrote an essay against the reform of weights and measures. This essay opened with a long scientific introduction by Vincenzo Flauti, Fergola's heir and leader of the synthetics. In it Flauti stated that mathematical abstractions are useless for addressing issues related to the public welfare. He used the standardization of weights and measures as an example of one of these useless measures. (Mazzotti, 2009, p. 265)

In 1838 Fortunato Padula (1815-1881), a graduate and later director of the School of Engineers, published a book entitled "Geometric problems resolved by algebraic analysis." In it he showed how to translate geometric problems into analytical language and how doing so simplified the solution process. His goal was to show the superiority of the analytical methods used by the *Scuola di Applicazione*. (Ferraro, 2013, p. 91) Flauti wanted to respond to the work of Padula. However, this wasn't easy given the current scientific production of the synthetic school. By the late 1830s most people had abandoned the synthetic method and Flauti himself was not publishing any new research. He realized there was a need for young scholars who could give it new vigor. He saw in Nicola Trudi (1811-1884) a new young genius who could revive the glories of the synthetic school. (Ferraro, 2013, p. 91) Trudi had recently found a very elegant, purely geometrical, solution to the problem of inscribing three circles in a triangle under specified conditions. His solution was published in the *Proceedings of Petersburg*. Flauti decided to issue a challenge to the analytics using this problem, recently solved so elegantly by Trudi, as one of the challenge items. In doing so Flauti would achieve two important goals: It would bring his young protégé Trudi to the attention of mathematicians, establishing Trudi in the research world. It also would raise the prestige of the synthetic school. (Ferraro, 2012, p. 6) Flauti chose two additional problems. The first of these involved inscribing in a given circle a triangle whose sides pass through three data points. Annibale Giordano, another protégé of the Fergola school, had solved this problem back in 1786 using synthetic methods. The third problem was a problem from projective geometry: To inscribe in a given pyramid four spheres that touch each other and touch the faces of the pyramid. Choosing problems that had been solved by the best students of the synthetic school along with a problem in projective geometry, a field of research of the synthetics, seemed to guarantee a win for the synthetic school. It also helped that the judges were members of the Academy of Sciences, which was under the control of the synthetic camp.

If Trudi was the principal researcher and star of the synthetic school then Fortunato Padula was his counterpart in the analytic school. In addition to his textbook he had written several works on geometry and mechanics. It was Padula who responded for the analytics to Flauti's challenge. Padula published a booklet dedicated to Carlo Rivera that opened with a historical piece in which he criticized and rejected all the theoretical assumptions of the synthetic school. He pointed out the limitations of the synthetic method, that the synthetic study of geometrical abstractions does not introduce students to the modern developments in mathematics. He then pointed out the advantages of the application of mathematics to arts and industries. (Mazzotti, 1998, p. 682) He solved the first two problems and states the third problem was unprovable; it was overdetermined.[i] While one could solve problems like this, he claimed that "we are not interested" in this kind of work. (Mazzotti, 1998, p. 681) Padula's solutions were rejected by the Academy of Sciences, who was in charge of judging the contest and awarding the prize. The synthesist Trudi was declared the official winner. Ironically, this 'win' ultimately became a defeat for the synthetic school. It turned out that Trudi's solutions were obtained using analytic methods; Trudi in fact was a master of analytic geometry. However, there was a temporary renewal of the synthetic school, thanks to Trudi. Projective geometry now became the main focus of his research. He also studied algebra, combinatorics and differential geometry. He did little research into the study of synthetic geometry. One may question whether he then really was the face of the next generation of Fergola's school.

Both Padula and Trudi's work attracted the attention of European mathematicians. (Ferraro, 2013, p. 98) Jacobi and Steiner, during their 1844 visit to Naples, praised both Padula and Trudi's work. Poncelet did the same in 1866. At the 1845 Italian Congresses of Science meeting in Naples, both Padula and Trudi lectured. Trudi spoke on algebraic equations, Padula on the theory of fluids. Mathematicians from all over Italy, as well as several other countries in Europe, attended the meeting. Naples was back in Europe; its mathematical isolation was at an end.

The story of Flauti and Trudi in many ways mirrors the story of Fergola and Giordano. The mentor took great delight in his pupil, promoting his accomplishments and supporting his research, even when that research took on a different focus than that of the teacher. But in the end both protégés turned their back on their teachers for a combination of political and mathematical reasons. Giordano joined Lauberg and the Jacobins. He saw the value that mathematics could play when applied to social and natural science. Trudi's defection was longer in coming. For the two decades following the public challenge of 1839, Flauti supported and encouraged Trudi's work in projective geometry. (Ferraro, 2012, p. 7) He considered Trudi a member of Fergola's school and Trudi's writings on projective geometry an integral part of the school's production. In doing so, Flauti showed a willingness to expand the interests of the school of Fergola beyond the geometry of the ancients or synthetic geometry. Of course Flauti himself, with his study of descriptive geometry, had similarly expanded and renewed the mathematics practiced by the synthetic school. (Ferraro, 2013, pp. 98-100) The synthetic school under Flauti continued to dominate the Academy of Sciences and the universities, but its influence began to wane. (Ferraro, 2013, p. 103)

In 1850 the universities underwent a reform that affected the teaching of mathematics. A chair of analytic geometry was added at the University of Naples. While the chair of geometry based on more synthetic and ancient methods was retained, the program initiated by Fergola was scaled down. A new generation of mathematicians was rising and these mathematicians came to think of themselves as Italian mathematicians. One of these was Emanuel Fergola, a distant relative of Nicola Fergola. Emanuel Fergola was a member of the synthetic school and, with Trudi, was entrusted with the publication of Nicola Fergola's course on integral and differential calculus. Emanuel Fergola's work in the 1850s was in the field of pure mathematics. In 1855 he became a professor at a military college and in 1860 a professor of calculus at the university.

The events of 1860 with the annexation of the Kingdom of the Two Sicilies (which included Naples) to Piedmont, leading to the unification of Italy, profoundly changed Neapolitan society and consequently its academic world. The old power groups reorganized themselves and wound up being incorporated into the new Italian state. Old mathematical disputes between the university and the *Scuola delle Applicazione* modified as the division between the work of the Corps of Engineers and that mathematics taught in the university began to close. There was a growing awareness of the link/close relationship between university teaching and research. (Ferraro, 2013, p. 105)

The relationship between Flauti and Trudi also changed as a result of the events of 1860. (Ferraro, 2012, p. 11) Flauti strongly opposed the annexation of Naples by Piedmont. While the rest of the population was celebrating the annexation, Flauti closed the balcony of his house in protest. He didn't stop there; he went on to write several pamphlets vehemently criticizing the reform laws of the University of Naples. These reforms were a continuation and expansion of the university reforms begun in 1850. With Italian unification in 1860, Flauti was able to retain the title of professor emeritus and Trudi retained his position as a university professor. Emanuel Fergola received a chair of sublime introduction to calculus. (Ferraro, 2013, p. 111)

However, when it came time to appoint members of the new Academy of Sciences to replace the deleted Academy of the Bourbons, Flauti, despite his many criticisms of the university reforms, was considered for membership. In the end, however, he was not chosen, but Trudi was chosen. (Ferraro, 2013, p. 114) It became apparent that it was not enough to be a good mathematician - political correctness was also required. Those associated with the Bourbon ruling class were offered the chance to join the new regime. Trudi chose to do so; Flauti did not. Trudi and Flauti parted ways. "Trudi became an Italian mathematician (and with him, Emanuel Fergola, Padula and Battaglini). Flauti, however, chose to die a Neapolitan." (Ferraro, 2012, p. 13) Flauti and Trudi did more than *part ways*. Flauti, feeling betrayed by Trudi, denounced him, accused him of ingratitude and hurled insults at him. Trudi's attitude was one of *forgetting*. In his subsequent writing he remembered the contributions of mathematicians such as Euler and Crelle but failed to mention those of Fergola. (Ferraro, 2013, p. 116) The adjective "Bourbon" took on a derogatory connotation and Flauti was definitely a Bourbon. In order to become an Italian mathematician, Trudi needed to break with his past. Flauti and the school of Fergola became for him a "*damnato memoriae*." Emanuel Fergola also aligned with the new order. He joined the wider mathematical community as a member of many institutes and European academies. In 1905 he became a senator of the United Kingdom of Italy.

Lessons

What can we learn from this period of Italian mathematics and its central players?

One of the first lessons is that changes in society have a definite impact on mathematics and how it is viewed and used, as well as the definition and role of a mathematician. The controversies between the synthetic and analytics school appeared to be about the right way to do mathematics, but underlying these controversies were major worldview assumptions about how truth is revealed, how mathematicians are to interact with society, the purpose of doing and teaching mathematics, and how to respond to changes in our natural world. For Fergola and his followers, mathematics and faith were definitely intertwined. Mathematics for them was a way to meditate on the wonders of God and his creative order. But mathematics became a tool to wage war against the changes they saw occurring in the world around them. In the early years Fergola was not only aware of the work of Lagrange and his analytical methods, but also advocated for these methods. That changed when these methods took on a political and social role that threatened the established order – the role of the church in society, the aristocratic power structure, and the local autonomy, which allowed for decisions to be made by leaders who understood the needs of the people in their care. He and his followers saw the effects the analytical methods were having in society. Many advocates for the analytical methods saw their abstraction and their separation from the mathematics of the geometric construction as a new way of looking at mathematics and its applications in society. Social and natural problems could be mathematized and solved. Gone was the spiritual element of mathematics; it was replaced by a sense of man's ability to solve all the problems in the world around him – social, economic, technological – with the help of mathematics. It was natural that Fergola would react to this. But he and his followers did it in such a way to set up a false dichotomy between the two kinds of mathematics. There wasn't a "right" kind of mathematics. The real argument was over what the mathematics represented and how it was used. Fergola's school also used mathematics to defend its worldview. By making claims that only a Christian could understand and produce good mathematics and using mathematics to justify certain religious or political practices, they used mathematics to promote their worldview, just as the analysts did.

It is interesting to note that Fergola and Flauti were both willing to expand their view of acceptable mathematics when it came to the work of their protégés. Fergola continued to support the work of Giordano even after Giordano joined the analytics; Flauti continued to support the work of Trudi even when Trudi's work focused more on analytic and projective geometry than on synthetic geometry. For both Fergola and Flauti this was probably due to the natural pride they had in the accomplishments of their students. For Flauti it also appeared to have been his desire to have the name of the Fergola school be glorified, so he 'saw' Trudi's work as a continuation of the Fergola tradition even though the mathematics Trudi did was different from that tradition. In the end it wasn't about the mathematics itself but what it represented to each party.

After the 1839 challenge, the importance of the synthetic school waned and most mathematicians abandoned its methods. Even the memory of the school was lost until late in the 19th century when Gino Loria (Loria, 1902) a mathematics historian, read a few lines about the school in some writing of Michael Chasles. (Ferraro, 2013, pp. 9-11) While the school itself was forgotten, the value of the synthetic method and the deductive method of Euclid resurfaced in the hands of Cremona, a brilliant mathematician and one of the most ardent revolutionists. (Ferraro, 2013, pp. 117-118)[ii] We will learn about the work of Cremona and other patriotic mathematicians in the second snapshot of this series. So while the synthetic method was advocated by Fergola's school as a reaction to the changes in Italian society that culminated in the unification of Italy, these same methods became one of the tools of the new government to educate its future leaders. Would Fergola rejoice or be abhorred?

Looking back on the history of mathematical controversies allows us to reflect with some detachment on decisions mathematicians make when dealing with changes in the world around them. Society and our understanding of the natural world are probably changing today at an even faster rate than in the 19th century. Mathematics plays a role in how society responds to those changes. It is important that we be clear about what the issues are and how mathematics actually applies to those issues. Rather than make false dilemmas (such as the *right* way to do geometry), mathematicians from different perspectives would be better off dialoguing and working together to find common ground. Of course, there may be times when it is necessary to take a firm stand, but these should be articulated clearly and not hidden behind false polemics.

In the second snapshot article of this series we look at Italian mathematicians from very different ideological perspectives who chose to transcend their differences and work toward the common goal of creating a national mathematical identity and to take their place in the European scientific community.

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[i] More equations than unknowns.

[ii] One of the first reforms undertaken in the new Kingdom of Italy was the reform of the educational system. Cremona worked on mathematics textbooks for the school. In Cremona's own research he used pure geometric methods entirely independent of algebra or analysis. His principal concern for the educational system was to prepare the future leaders of the country. He felt this could best be done by teaching them methods to deal effectively with problems, to learn *how* to think rather than just acquiring a mass of knowledge.

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